

# N-person quantum Russian roulette

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We generalize the concept of quantum Russian roulette introduced in [A.G.M. Schmidt, L. da Silva, *Physica A* **392** (2013) 400-410]. Our model coincides with the previous one in the case of the game with two players and gives the suitable quantum description for any finite number of players.

## 1. Introduction

In order to model problems involving conflicts among individuals von Neumann and Morgenstern [1] introduced the mathematical concept of games. These ideas found applications in several fields of science, from Economics to Social Sciences and reached the quantum domain back in 1999, when Meyer [2] and Eisert, Wilkens and Lewenstein [3] elaborated quantum versions of a coin-flip game and of the prisoner dilemma (PD), respectively. The classical PD game is the keystone for modelling cooperative behaviour between animals, economic agents, strategies for iterated games [4] and even some RNA virus[5]. Quantum mechanics can solve such dilemma, as were shown in reference [3], if one player could use a quantum strategy — quantum in the sense of an strategy that is not cooperate nor defect. Experimentally the PD was realized using NMR by Du and co-workers [6] and recently using optical techniques by Pinheiro *et al* [7]. Several papers followed these two seminal works, we can mention Monty-Hall problem [8], discoordination games [9], repeated quantum games [10, 11], multiplayer quantum games [12], quantum auctions [13], quantum dating market [14] and minority game both theoretically [15] and experimentally [16].

On the other hand, game theory can model also not only cooperative behaviour but conflict situations. This is the case of the so-called quantum

duel studied by Flitney and Abbott [17] and revisited by Schmidt and Paiva [18]. In this game each player, Alice and Bob for instance, has a qubit  $|\psi\rangle = c_1|1\rangle + c_2|0\rangle$  where  $|1\rangle$  represents the “alive” state and  $|0\rangle$  the “dead” state, and their only objective is to flip his/hers opponent’s spin. Following this idea Schmidt and da Silva proposed the quantum version of the gamble known as Russian roulette, where players shoot themselves at point blank range using a gun loaded with just one bullet [19]. The authors found some interesting results concerning the cases where the gun was fully loaded as well as when there was just one quantum bullet inside its chambers. However, in the case of 3-person game, one can find only sketch of a possible form of the scheme rather than the exact protocol. The framework, however, can lead to quite different models, and some of them inconsistent with the 2-person quantum roulette. The previous analysis suggests a possibility of studying the game with larger number of players as the Authors construct some particular case of quantum description of the 3-person. Our research is aimed at showing that there exists a natural extension of the two-person quantum Russian roulette to the  $n$ -person case.

A formal theory for quantum games was initiated with the concepts of quantum matrix games [3, 20]. However, quantum methods have found application in many other types of games and decision problems from then on. Apart from the quantum scheme for extensive games [21] one can find such quantized problems as the quantum Monty Hall problem [22, 8, 23] and the quantum roulette [24, 25]. The essential for our studying are ones concerning quantum duels and truels [17, 18]. It was shown in paper [19] that the schemes for quantum duels and truels can be adapted quantum Russian roulette where the Authors provide us with the concept for two and three-person case.

The outline for our paper is the following: in section 2 we review the two-person quantum Russian roulette. Section 3 is devoted to present the generalized operators for the  $n$ -person case and to discuss some illustrative examples. In the final section we conclude the work.

## 2. Two-person quantum Russian roulette

According to [19] the scheme for 2-person quantum Russian roulette is described as follows: each player has a qubit and the game is played in a Hilbert space  $\mathbb{C}^2 \otimes \mathbb{C}^2$ . The game begins with both players alive, namely  $|\psi_0\rangle = |11\rangle$ , where  $i$ -th qubit belongs to  $i$ -th player for  $i = 1, 2$ . The referee prepares operators  $O_1$  and  $O_2$  (the operators are also labelled  $O_i(\gamma_j)$  if the

angle that the operator depends on is relevant),

$$\begin{aligned} O_1 &= [e^{-i\alpha} \sin(\gamma/2)|11\rangle + ie^{i\beta} \cos(\gamma/2)|01\rangle]\langle 11| \\ &\quad + [e^{i\alpha} \sin(\gamma/2)|01\rangle + ie^{-i\beta} \cos(\gamma/2)|11\rangle]\langle 01| + |00\rangle\langle 00| + |10\rangle\langle 10|; \\ O_2 &= [e^{-i\alpha} \sin(\gamma/2)|11\rangle + ie^{i\beta} \cos(\gamma/2)|10\rangle]\langle 11| \\ &\quad + [e^{i\alpha} \sin(\gamma/2)|10\rangle + ie^{-i\beta} \cos(\gamma/2)|11\rangle]\langle 10| + |00\rangle\langle 00| + |01\rangle\langle 01|. \end{aligned} \quad (1)$$

which the first and the second player act on the initial state. The game proceeds in rounds (the referee is allowed to settle any number of rounds). The state after  $k$  rounds,  $k = 1, 2, \dots, m$  is defined recursively,

$$\begin{aligned} |\psi_0\rangle &= |11\rangle, \\ |\psi_k\rangle &= O_2(\gamma_{2k})O_1(\gamma_{2k-1})|\psi_{k-1}\rangle, \end{aligned} \quad (2)$$

and the outcome after  $k$ -th round is given by a measurement with respect to the computational basis,

$$\langle S \rangle = \sum_{i,j=0,1} s_{ij} |\langle ij|\psi_k\rangle|^2, \quad (3)$$

where  $s_{ij}$  is the outcome corresponding to the measured state  $|ij\rangle$ . In particular, the measurement can be identified with a payoff function. For example, in [19], the Authors studied the quantum Russian roulette with payoff functions

$$\begin{aligned} \langle \$1 \rangle &= \frac{1}{2} [1 + |\langle 10|\psi_m\rangle|^2 - |\langle 01|\psi_m\rangle|^2 - |\langle 00|\psi_m\rangle|^2]; \\ \langle \$2 \rangle &= \frac{1}{2} [1 + |\langle 01|\psi_m\rangle|^2 - |\langle 10|\psi_m\rangle|^2 - |\langle 00|\psi_m\rangle|^2]. \end{aligned} \quad (4)$$

which means that each of the players wins 1 if he is only one who survived and 1/2 if both players are alive. Otherwise, they win nothing. Note that, other payoff functions can be defined as well. For example, the case that a player receives payoff 1 only when he is the only survivor and the other player 'receives' -1 is equally natural, especially with reference to the classical game, where the draw is not normally possible, i.e.,

$$\langle \$1 \rangle = |\langle 10|\psi_m\rangle|^2 - |\langle 01|\psi_m\rangle|^2, \quad \langle \$2 \rangle = |\langle 01|\psi_m\rangle|^2 - |\langle 10|\psi_m\rangle|^2. \quad (5)$$

Scheme (1)-(3) generalizes the classical 2-person Russian roulette with a gun having two chambers in the barrel. Indeed, let us assume that  $\gamma_i = 0$  represents a bullet in the player  $i$ 's chamber and  $\gamma_i = \pi$  means that  $i$ -th

chamber is empty. Then a result corresponding to each of four possible cases in the classical game, that is, when the barrel of the gun is full, a bullet is in the first or the second chamber and the barrel is empty, can be reconstructed with the results of scheme (1)-(3) for  $(\gamma_1, \gamma_2) \in \{(0, 0), (0, \pi), (\pi, 0), (\pi, \pi)\}$ . From (2) we have

$$\begin{aligned} O_2(0)O_1(0)|11\rangle &= ie^{i\beta}|01\rangle, & O_2(0)O_1(\pi)|11\rangle &= ie^{i(-\alpha+\beta)}|10\rangle; \\ O_2(\pi)O_1(0)|11\rangle &= ie^{i\beta}|01\rangle, & O_2(\pi)O_1(\pi)|11\rangle &= e^{-2i\alpha}|11\rangle, \end{aligned} \quad (6)$$

Thus, if the states  $|0\rangle$  and  $|1\rangle$  are assumed to represent *dead* and *alive* state, respectively, the measurement outcome of each of the resulting states (6) identifies with the classical results. For example, if the bullet is inside the second player's chamber, the game ends with the death of the second player. In the quantum case it means that operators (1) are prepared with  $(\gamma_1, \gamma_2) = (\pi, 0)$ , and the measurement on  $O_2(0)O_1(\pi)|11\rangle$  outputs  $|10\rangle$ .

It worth noting that, in general, scheme (1)-(3) takes into consideration the case in which a bullet in each chamber  $i$  may be found with some probability  $p_i$ . Indeed, the final state associated with the general form of operators (1) takes on the form

$$\begin{aligned} O_2(\gamma_2)O_1(\gamma_1)|11\rangle &= ie^{i\beta} \cos(\gamma_1/2)|01\rangle \\ &+ ie^{-i(\alpha-\beta)} \sin(\gamma_1/2) \cos(\gamma_2/2)|10\rangle + e^{-2i\alpha} \sin(\gamma_1/2) \sin(\gamma_2/2)|11\rangle \end{aligned} \quad (7)$$

The result given by the measurement (3) on state (7) coincides with the classical result if we assume  $p_i \equiv \cos^2(\gamma_i/2)$ . For example, the case where the first player is alive and the second player fails at the end of the first round takes place with probability  $(1-p_1)q_1$ , and the quantum counterpart  $|10\rangle$  is measured with probability  $\sin^2(\gamma_1/2) \cos^2(\gamma_2/2)$ .

### 3. N-person quantum Russian roulette

Now, we extend scheme (1)-(3) to consider the game with more than two players. We stick to the Russian roulette in which the referee prepares the gun with one chamber for each player, and a player finds a bullet in his own chamber with some probability.

The previous section shows that the proper definition of 2-person quantum Russian roulette lies in a construction of suitable operators (1). Now, we use the same reasoning for the  $n$ -person game. Let us denote by  $\mathbb{1}$  and  $U_i$  the unitary operators on  $\mathbb{C}^2$  given by formulae

$$\begin{aligned} \mathbb{1} &= |0\rangle\langle 0| + |1\rangle\langle 1|; \\ U_i &= \sin(\gamma_i/2)(e^{i\alpha}|0\rangle\langle 0| + e^{-i\alpha}|1\rangle\langle 1|) + i \cos(\gamma_i/2)(e^{i\beta}|0\rangle\langle 1| + e^{-i\beta}|1\rangle\langle 0|). \end{aligned} \quad (8)$$

Then operators (1) can be written as follows:

$$O_1 = \mathbb{1} \otimes |0\rangle\langle 0| + U_1 \otimes |1\rangle\langle 1|, \quad O_2 = |0\rangle\langle 0| \otimes \mathbb{1} + |1\rangle\langle 1| \otimes U_2, \quad (9)$$

The first term of each  $O_i$  leaves the state unchanged if both players are initially dead ( $|00\rangle$ ) or only player  $i$  is alive ( $|10\rangle$ ). Otherwise, the second term performs unitary operation on player  $i$ 's qubit leaving the opponent's qubit unchanged. It can be done the same in  $n$ -person case. That is, player  $i$ 's operator  $O_i$  does nothing if either only player  $i$  is alive or all the players are dead, and  $O_i$  performs unitary transformation only on player  $i$ 's qubit.

**Definition 3.1** *An  $n$ -person quantum Russian roulette with  $m$  rounds is defined by the following components:*

- the players' operators

$$\begin{aligned} O_1 &= \mathbb{1} \otimes (|0\rangle\langle 0|)^{\otimes n-1} + \sum_{\substack{(i_1, \dots, i_{n-1}) \in \{0,1\}^{n-1} \\ (i_1, \dots, i_{n-1}) \neq (0, \dots, 0)}} U_1 \otimes |i_1 \dots i_{n-1}\rangle\langle i_1 \dots i_{n-1}|; \\ O_2 &= |0\rangle\langle 0| \otimes \mathbb{1} \otimes (|0\rangle\langle 0|)^{\otimes n-2} + \sum_{\substack{(i_1, \dots, i_{n-1}) \in \{0,1\}^{n-1} \\ (i_1, \dots, i_{n-1}) \neq (0, \dots, 0)}} |i_1\rangle\langle i_1| \otimes U_2 \otimes |i_2 \dots i_{n-1}\rangle\langle i_2 \dots i_{n-1}|; \\ &\vdots \\ O_n &= (|0\rangle\langle 0|)^{\otimes n-1} \otimes \mathbb{1} + \sum_{\substack{(i_1, \dots, i_{n-1}) \in \{0,1\}^{n-1} \\ (i_1, \dots, i_{n-1}) \neq (0, \dots, 0)}} |i_1 \dots i_{n-1}\rangle\langle i_1 \dots i_{n-1}| \otimes U_n. \end{aligned} \quad (10)$$

- the final state  $|\psi_k\rangle$  at  $k$ -th round,  $k = 1, \dots, m$ ,

$$\begin{aligned} |\psi_0\rangle &= |1\rangle^{\otimes n} \\ |\psi_k\rangle &= O_n(\gamma_{kn}) \cdots O_2(\gamma_{(k-1)n+2}) O_1(\gamma_{(k-1)n+1}) |\psi_{k-1}\rangle \end{aligned} \quad (11)$$

- the measurement after  $k$  round,

$$S = \sum_{(i_1, \dots, i_n) \in \{0,1\}^n} s_{i_1, \dots, i_n} |\langle i_1, \dots, i_n | \psi_k \rangle|^2. \quad (12)$$

The clear way of describing (10) allows us to easily prove that

**Proposition 3.1.1** *Operators defined by (10) are unitary.*

*Proof.* Without loss of generality we prove that  $O_1$  is a unitary on  $(\mathbb{C}^2)^{\otimes n}$ . Since  $\mathbb{1}$  and  $U_1$  are unitary operators we obtain

$$O_1^\dagger O_1 = O_1 O_1^\dagger = \mathbb{1} \otimes (|0\rangle\langle 0|)^{\otimes n-1} + \sum_{\substack{(i_1, \dots, i_{n-1}) \in \{0,1\}^{n-1} \\ (i_1, \dots, i_{n-1}) \neq (0, \dots, 0)}} \mathbb{1} \otimes |i_1 \dots i_{n-1}\rangle\langle i_1 \dots i_{n-1}|. \quad (13)$$

But

$$\sum_{\substack{(i_1, \dots, i_{n-1}) \in \{0,1\}^{n-1} \\ (i_1, \dots, i_{n-1}) \neq (0, \dots, 0)}} \mathbb{1} \otimes |i_1 \dots i_{n-1}\rangle\langle i_1 \dots i_{n-1}| = \mathbb{1}^{\otimes n} - \mathbb{1} \otimes (|0\rangle\langle 0|)^{\otimes n-1}, \quad (14)$$

which implies that  $O_1^\dagger O_1 = O_1 O_1^\dagger = \mathbb{1}^{\otimes n}$ .  $\square$

The scheme defined by (10)-(12) generalizes a Russian roulette in a similar way as scheme (1)-(3) does.

**Example 3.2** Let us consider a 3-person quantum Russian roulette with a single round given by definition 3.1. Putting  $n = 3$  and  $k = 1$  into (10) and (11) we obtain the following final state:

$$\begin{aligned} |\psi_1\rangle = O_3 O_2 O_1 |111\rangle = & -e^{2\beta i} \cos(\gamma_1/2) \cos(\gamma_2/2) |001\rangle \\ & -e^{(-\alpha+2\beta)i} \cos(\gamma_1/2) \sin(\gamma_2/2) \cos(\gamma_3/2) |010\rangle \\ & +ie^{(-2\alpha+\beta)i} \cos(\gamma_1/2) \sin(\gamma_2/2) \sin(\gamma_3/2) |011\rangle \\ & -e^{(-\alpha+2\beta)i} \sin(\gamma_1/2) \cos(\gamma_2/2) \cos(\gamma_3/2) |100\rangle \\ & +ie^{(-2\alpha+\beta)i} \sin(\gamma_1/2) \cos(\gamma_2/2) \sin(\gamma_3/2) |101\rangle \\ & +ie^{(-2\alpha+\beta)i} \sin(\gamma_1/2) \sin(\gamma_2/2) \cos(\gamma_3/2) |110\rangle \\ & +e^{-3i\alpha} \sin(\gamma_1/2) \sin(\gamma_2/2) \sin(\gamma_3/2) |111\rangle. \end{aligned} \quad (15)$$

Bearing in mind that  $\gamma_i$  equal to 0 and  $\pi$  are identified with an empty and full chamber of player  $i$ , respectively, and state  $|0\rangle$  represents *dead* state whereas  $|1\rangle$  represents *alive* state, let us examine the four possible deterministic cases:

- *No bullets* An empty barrel in the gun makes all the players alive in the classical game. The quantum counterpart is accomplished then by taking  $(\gamma_1, \gamma_2, \gamma_3) = (\pi, \pi, \pi)$  in scheme (10)-(12) as the final state  $|\psi_1\rangle$  takes on the form  $O_1(\pi)O_2(\pi)O_3(\pi)|111\rangle = e^{-3i\alpha}|111\rangle$ .
- *One bullet* The game with exactly one bullet in chamber  $i$  always ends with the dead of player  $i$  in the classical case. The same is

obtained by means of the quantum scheme. For example, if the second player has got a bullet i.e.  $(\gamma_1, \gamma_2, \gamma_3) = (\pi, 0, \pi)$ , the state  $|101\rangle$  since the final state is  $O_1(\pi)O_2(0)O_3(\pi)|111\rangle = ie^{(-2\alpha+\beta)i}|101\rangle$ .

- *Two bullets* There are two ways to play the 3-person Russian roulette, where at most one chamber in the barrel is left empty. The game can be playing until either one of the players shoots himself or only one player stays alive. Definition 3.1 concerns the latter case. For example, if player 1's chamber is empty while the other players have got bullets in their chambers, the first player is the only survivor in the end of the classical game. The same occurs in the quantum case since  $(\gamma_1, \gamma_2, \gamma_3) = (\pi, 0, 0)$  implies the measurement result  $|100\rangle$ .
- *Three bullets* If  $(\gamma_1, \gamma_2, \gamma_3) = (0, 0, 0)$ , i.e., the gun is fully loaded, the scheme given by definition 3.1 also imitates the classical game as the final state is  $-e^{2\beta i}|001\rangle$ . Since the third player is the last to pull the trigger and his opponents' chambers are full, player 3 becomes the only survivor just before his turn to fire the gun, and therefore he wins the game.

Scheme (10)-(12) generalizes the Russian roulette game even if the referee prepares bullets in the gun in a stochastic way, that is, if player  $i$  finds a bullet in his chamber with probability  $p_i$ . Similarly to the 2-person case if it assumed that  $p_i \equiv \cos(\gamma_i/2)$ , the results of the measurement on (15) are consistent with the results of the classical game.

Like in the case of quantum duels and truels, if  $|\psi_0\rangle = |1\rangle^{\otimes n}$ , it is not possible to obtain superior results in the quantum Russian roulette playing only one round. However, if the game is played two or more rounds (without measuring state  $|\psi_k\rangle$  before the last round is played), the results in the quantum game can differ significantly from the corresponding results in the classical game.

**Example 3.3** To reduce necessary calculations, let us make an assumption that the players play two rounds and the referee prepares a gun in which each player finds a bullet in their chamber with equal probability. In other words, following formula (11), for  $i = 1, 2, 3$ , player  $i$  manipulates with  $O_i(\gamma_i)$  and  $O_i(\gamma_{i+3})$  such that  $\gamma_i, \gamma_{i+3} = \pi/2$ .

Let us ask a question: what is the average probability that all the players remain alive after two rounds. As probability of measuring state  $|111\rangle$  after two rounds is given by  $|\langle 111|\psi_2\rangle|^2$  in the quantum case, it is sufficient to determine only the amplitude  $a_{111}$  associated with basis state  $|111\rangle$  instead of the whole state  $|\psi_2\rangle$ . The amplitude takes on the following form:

$$a_{111} = \frac{e^{-i\alpha}}{2\sqrt{2}} + \frac{1}{8} (e^{-2i\alpha} - 3e^{-4i\alpha} + e^{-6i\alpha}) \quad (16)$$

Now, if we make a natural assumption that  $\alpha$  and  $\beta$  are uniformly distributed, the expected probability  $E[|\langle 111|\psi_2\rangle|^2]$  is given by

$$E[|\langle 111|\psi_2\rangle|^2] = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} |a_{111}|^2 d\alpha d\beta = \frac{19}{64}. \quad (17)$$

To compare result (17) with the classical case, note that the measurement on state  $|\psi_1\rangle$  was not performed. Thus, we should compare (17) only with a classical probability of the result of the second round, (given that the result of the first round allows players to play one more time), and such a probability is equal to  $1/8$ . Thus, in the quantum domain, the players all are more than twice as likely to survive the second round.

In order to make another comparison of quantum Russian roulette with the classical one, let us consider that not only  $\alpha$  and  $\beta$  parameters are uniformly distributed, but all angles  $\gamma_i$  are also uniformly distributed. Now the probability for our 3-person two rounds, quantum game is given by,

$$P_{3,2} = \frac{1}{4\pi^8} \int_0^{2\pi} \int_0^{2\pi} d\alpha d\beta \int_0^\pi \int_0^\pi \int_0^\pi \int_0^\pi \int_0^\pi \int_0^\pi |\langle 111|\psi_2\rangle|^2 d\gamma_1 \cdots d\gamma_6. \quad (18)$$

One can investigate similar problems when there are 4- and 5-persons in the gamble, which yield 8-fold and 10-fold angular integrals analogous to (18). Our results are summarized in the table 1 and one can infer that the quantum version produces greater probabilities for an “all alive” outcome.

**Table 1** *Probabilities for all players being alive after the second round when parameters  $\alpha$  and  $\beta$  as well as angles  $\gamma_i$  are uniformly distributed. Quantum version produces greater probabilities.*

<i>Number of players</i>	<i>Quantum</i>	<i>Classical</i>
3	$\frac{1}{8} + \frac{1}{64} + \frac{7}{2\pi^4}$	$\frac{1}{8}$
4	$\frac{1}{16} + \frac{1}{256} + \frac{6}{\pi^8} + \frac{23}{8\pi^4}$	$\frac{1}{16}$
5	$\frac{1}{32} + \frac{1}{1024} + \frac{15}{\pi^8} + \frac{77}{32\pi^4}$	$\frac{1}{32}$

**Example 3.4** Another interesting example, concerning the 3-person quantum Russian roulette, takes place when the referee is unfair and prepares the gun with no bullets for two players, and the third players’ chambers are fully loaded. The first round obviously reproduces the classical result since the player shoot himself and ends in the dead state; the other two players remain in the alive state. This dead state can be flipped back to  $|1\rangle$  in the



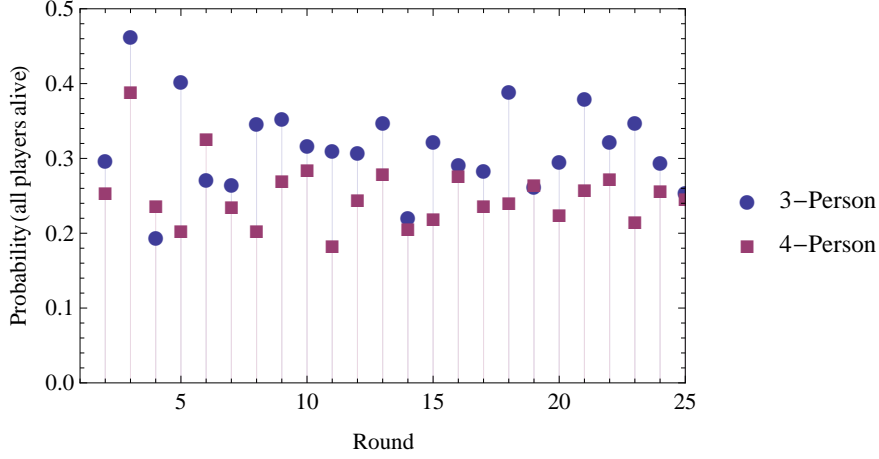


Fig. 1. Plot of the probabilities  $|\langle a_{111}|\psi_n\rangle|^2$  and  $|\langle a_{1111}|\psi_n\rangle|^2$  as a function of the number of rounds  $n$  as studied in example 3.5.

second round — in all three cases namely  $\gamma_1 = \gamma_4 = 0$  or  $\gamma_2 = \gamma_5 = 0$  or  $\gamma_3 = \gamma_6 = 0$  — the final state is proportional to  $|111\rangle$ , i.e., all player end alive with probability equal to unity. The third round yields the same classical result as well as the fourth round return to the second one and so forth. A player with fully loaded chambers kills himself after an odd number of rounds if final measurement takes place, and remains alive if an even number of rounds were played.

**Example 3.5** As a final example let us study what would happen if the game was played several times, e.g., 25 rounds. Firstly let us consider the case similar to example 3.3 where players have chance of 50% of having a bullet inside their chambers. One observes in figure 1 that the maximum probability takes place in the third round and does not exceed 48%. On the other hand, we infer from figure 2 that the last player to shot is the one who is more likely to survive alone, namely his average chance is 10.6%. Analogous conclusions can be inferred for the 4-person game where the maximum probability for an “all alive” outcome is around 40% and occurs at the third round, see figure 3; as well as the last player has a slightly greater chance of surviving alone, i.e., 6.2%. In both cases the second player to shot is the one who has worst probabilities of defeating his opponents: 7.7% and 3.9% for 3-person and 4-person quantum Russian roulette respectively. Conversely, if all the players have only one bullet inside their chambers, and that each bullet is smeared out uniformly inside each chamber, i.e., in a  $n$ –round game each player has  $\gamma_n = 2 \arccos \frac{1}{\sqrt{n}}$ . Integrating over  $\alpha$

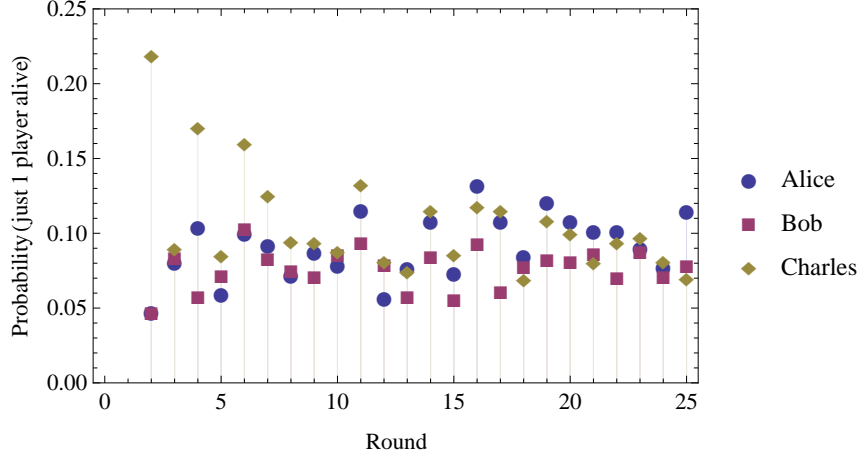


Fig. 2. Plot of the probabilities  $|\langle a_{100} | \psi_n \rangle|^2$  (Alice),  $|\langle a_{010} | \psi_n \rangle|^2$  (Bob) and  $|\langle a_{001} | \psi_n \rangle|^2$  (Charles) as a function of the number of rounds  $n$  as studied in example 3.5.

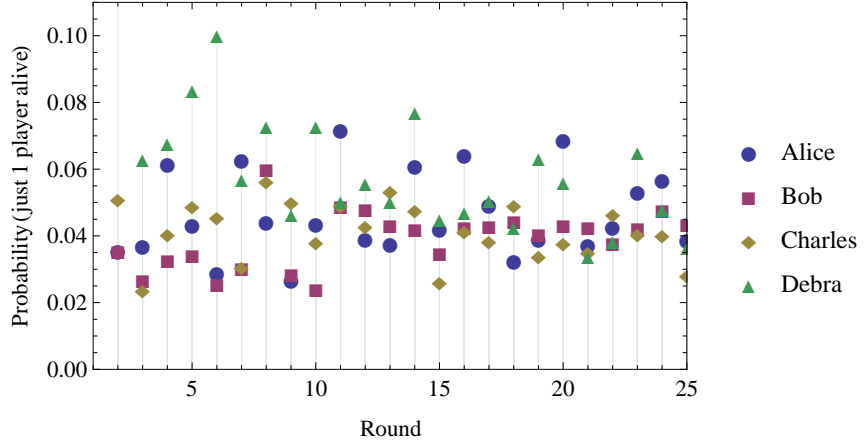


Fig. 3. Plot of the probabilities  $|\langle a_{1000} | \psi_n \rangle|^2$  (Alice),  $|\langle a_{0100} | \psi_n \rangle|^2$  (Bob),  $|\langle a_{0010} | \psi_n \rangle|^2$  (Charles) and  $|\langle a_{0001} | \psi_n \rangle|^2$  (Debra) as a function of the number of rounds  $n$  as studied in example 3.5.

and  $\beta$  one observes that the probability of surviving increases — for 3- and 4-person the these probabilities exceed 80% — after each round since the players have smaller chance of firing themselves, a problem analogous to the well-known bomb-quest [26].

#### 4. Conclusion

We generalized the two-person quantum Russian roulette game for an arbitrary number of players. Our operators (10) allow for any preparation of the gun. We applied our results to four representative examples and compared the quantum game with the classical one. In general the quantum game yields better outcomes for the players.

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